Name: ______________________ GTID: ______________________

- Fill out your name and Georgia Tech ID number.
- This quiz contains 3 pages. Please make sure no page is missing.
- The grading will be done on the scanned images of your test. Please write clearly and legibly.
- Answer the questions in the spaces provided. We will scan the front sides only by default. If you run out of room for an answer, continue on the back of the page and notify the TA when handing in.
- Please write detailed solutions including all steps and computations.
- The duration of the quiz is 30 minutes.

Good luck!

1. (33 points) Determine the longest interval in which the given initial value problem

$$ty'' + 3y = t, y(1) = 1, y'(1) = 2$$

is certain to have a unique twice differentiable solution. You do not need to find the solution.

Solution: Rewriting the equation to

$$y'' + \frac{3}{t}y = 1$$

leads to $p(t) = 0$ and $q(t) = \frac{3}{t}$. Note that $q(t)$ is discontinuous at $t = 0$.
The interval where $p(t)$ and $q(t)$ are continuous and containing $t_0 = 1$ is $(0, \infty)$. Hence, the maximal interval in which a unique solution exists is $(0, \infty)$.

2. (33 points) Find the general solution for

$$y'' - 3y' + 2y = 0.$$

Solution: The corresponding characteristic equation is $\lambda^2 - 3\lambda + 2 = 0$ and its solutions are $\lambda_1 = 1$ and $\lambda_2 = 2$. Therefore the general solution is

$$y(t) = c_1 e^t + c_2 e^{2t},$$

for constants $c_1, c_2 \in \mathbb{R}$, due to the scheme we explained in class of relating a second order ODE to a system of linear first order ODE.

3. (34 points) Solve

$$x^2 y''(x) + xy'(x) + 4y(x) = 0$$

when $x > 0$.

Solution: By substituting $t = \ln x$ and $Y(t) = y(e^t) = y(x)$, we see that

$$Y'(t) = e^t y'(e^t) = xy'(x),$$

and

$$Y''(t) = e^{2t} y''(e^t) + e^t y'(e^t) = x^2 y''(x) + xy'(x),$$

and therefore $x^2 y''(x) + xy'(x) + 4y(x) = 0$ is equivalent to

$$Y''(t) + 4Y(t) = 0.$$

In order to solve the above, we solve the corresponding characteristic equation: $\lambda^2 + 4 = 0$.

Hence, $\lambda = \pm 2i$

So, $Y_c(t) = c_1 \cos(2t) + c_2 \sin(2t)$.

Back substituting $t = \ln x$ yields,

$$y_c(x) = c_1 \cos(2 \ln x) + c_2 \sin(2 \ln x).$$